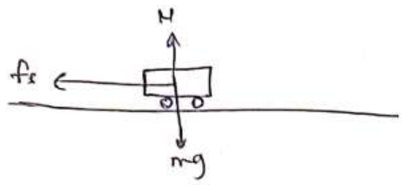
(1) * Rotational Dynamics *

* Vehicle along horizontal circular Track:

→ consider a vehicle of mass 'm' moving along a horizontal circular track of radius 'r'.

∴ force acting on the vehicle are:

1. Weight (mg) vertically downward.
2. Normal reaction (N) vertically upward.
3. force of static friction bet the road & tires. (fs).



Normal reaction 'N' balance weight 'mg'

$\therefore N = mg \longrightarrow (1)$

'fs' provides centripetal force.

$\therefore f_s = \frac{mv^2}{r} \longrightarrow (2)$

divide (2) by (1)

$\frac{f_s}{N} = \frac{\frac{mv^2}{r}}{mg}$

$\because \frac{f_s}{N} = \mu_s$

$\therefore \mu_s = \frac{v^2}{rg}$

$rg\mu_s = v^2$

$\therefore v^2 = \mu rg$

$\boxed{v_{max} = \sqrt{\mu rg}}$

Here r & g are constant.

$v \propto \sqrt{\mu_s}$

The vehicle value of μ_s cannot be inc↑ as it causes wear & tear of tyres of vehicle.

Also, value of μ_s is not reliable becz it goes on decr as it rain or due to oil spill.

∴ Roads are banked.

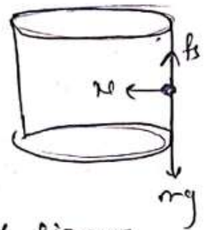
Q> Define Banking of road stat its Meaning.

"The phenomenon of raising the outer edge of road over its inner edge by certain angle with the horizontal is called as 'Banking of road'".

* Wall of death:

It is a cylindrical wall inside which a vehicle perform horizontal circle.

⇒ consider a vehicle of mass 'm' performing horizontal circle of radius 'r' inside a wall of death. When the vehicle is in given position as shown in the above figure



∴ The force acting on the vehicles are:

1. Normal reactⁿ (N) ⊥ar to the surface.
2. 'mg' vertically downward.
3. fs acting vertically upward. (fs against the direction of motion)

∵ fs balance mg.

$\therefore f_s = mg \longrightarrow (1)$

'N' provides centripetal force

$N = \frac{mv^2}{r} \longrightarrow (2)$

(2) by (1)

$\frac{N}{f_s} = \frac{\frac{mv^2}{r}}{mg}$

$\frac{N}{f_s} = \frac{v^2}{rg}$

But, $\frac{f_s}{N} = \mu_s \Rightarrow f_s = \mu_s N.$

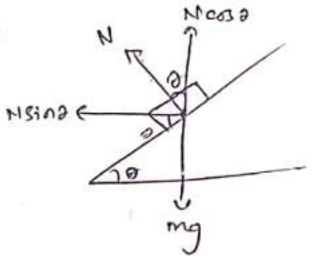
$\therefore \frac{N}{\mu_s N} = \frac{v^2}{rg}$

$\frac{rg}{\mu_s} = v^2$

$\boxed{\therefore v_{min} = \sqrt{\frac{rg}{\mu_s}}}$

* Vehicle on a Banked Road:

→ consider a vehicle of mass 'm' moving along a curved road with radius of curvatur 'r' banked at angle 'θ'



∴ The force acting on the vehicll are;

① mg vertically downwad.

② 'N' $\perp^{lar}$ to the surface

③ 'N' resolved into,

(i) Ncosθ vertically component

(ii) Nsinθ horizontal component.

∴ mg balance Ncosθ

$$\therefore mg = N\cos\theta \quad \text{—①}$$

'Nsinθ' provides centripital force.

$$\therefore \frac{mv^2}{r} = N\sin\theta \quad \text{—②}$$

② by ①

$$\frac{\frac{mv^2}{r}}{mg} = \frac{N\sin\theta}{N\cos\theta}$$

$$\frac{v^2}{rg} = \tan\theta \Rightarrow \boxed{\theta = \tan^{-1}\left(\frac{v^2}{rg}\right)}$$

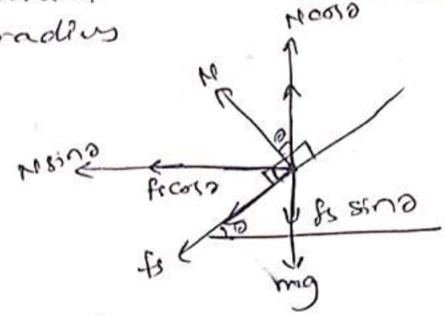
$$v^2 = rg\tan\theta$$

$$\boxed{v = \sqrt{rg\tan\theta}}$$

* Drive an expression for maximum speed of a vehicle on a Banked Road:

max/upper
↳ fs → lower.
↳ ⊖ niche ayega

⇒ consider a vehicle of mass 'm' moving along a horizontal curved road with radius of curvature 'r' banked at angle 'θ'.



The force acting on a vehicle are;

① mg vertically downward

② 'N' $\perp^{lar}$ to the surface

∴ 'N' resolved into.

(a) Ncosθ → vertical component

(b) N sinθ → horizontal componet.

③ fs against the direction of motion

∴ fs resolved into

(a) fs sinθ → vertical component.

(b) fs cosθ → horizontal component

Ncosθ balance mg & fs sinθ

$$\therefore N\cos\theta = mg + f_s\sin\theta$$

$$\therefore mg = N\cos\theta - f_s\sin\theta \quad \text{—①}$$

N sinθ & fs cosθ provids centripital force

$$\therefore N\sin\theta + f_s\cos\theta = \frac{mv^2}{r} \quad \text{—②}$$

② by ①

$$\frac{\frac{mv^2}{r}}{mg} = \frac{N\sin\theta + f_s\cos\theta}{N\cos\theta - f_s\sin\theta}$$

$$\frac{v^2}{rg} = \frac{\frac{N\sin\theta}{N\cos\theta} + \frac{f_s\cos\theta}{N\cos\theta}}{\frac{N\cos\theta}{N\cos\theta} - \frac{f_s\sin\theta}{N\cos\theta}} = \frac{\tan\theta + \frac{f_s}{N}}{1 - \frac{f_s}{N}\tan\theta}$$

If $v = v_{max}$ ∴ $f_s = \mu_s N$. ∴ $\frac{f_s}{N} = \mu_s$.

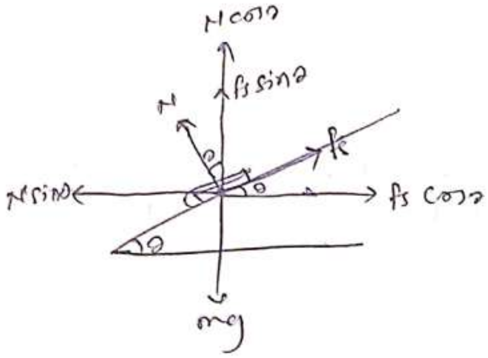
(2)

$\therefore \frac{V_{max}^2}{rg} = \frac{\tan\theta + \mu}{1 - \mu_s \tan\theta}$

$V_{max}^2 = rg\left(\frac{\tan\theta + \mu_s}{1 - \mu_s \tan\theta}\right)$

$$\boxed{V_{max} = \sqrt{rg\left(\frac{\tan\theta + \mu_s}{1 - \mu_s \tan\theta}\right)}}$$

* Drive an expression for minimum speed limit of a vehicle on a banked road:



⇒ consider a vehicle of mass 'm' moving along a curved road of radius of curvature 'r' banked at angle 'θ'.

Force acting on the vehicles are:

1. mg vertically downward
2. N $\perp$ to the surface
   ∴ N resolved into
   (a) vertical compont $N\cos\theta$
   (b) horizontal compont $N\sin\theta$.
3. f_s against the direction of motion
   ∴ It resolved into
   (a) vertical component $f_s\sin\theta$
   (b) horizontal compont $f_s\cos\theta$.

mg balance $f_s\sin\theta$ & $N\cos\theta$

$\therefore mg = N\cos\theta + f_s\sin\theta$ — (1)

$N\sin\theta$ & $f_s\cos\theta$ provides centripital force.

$N\sin\theta - f_s\cos\theta = \frac{mv^2}{r}$ — (2)

(2) by (1)

$\frac{\frac{mv^2}{r}}{mg} = \frac{N\sin\theta - f_s\cos\theta}{N\cos\theta + f_s\sin\theta}$

Divide by $N\cos\theta$.

$\frac{v^2}{rg} = \frac{\frac{N\sin\theta}{N\cos\theta} - \frac{f_s\cos\theta}{N\cos\theta}}{\frac{N\cos\theta}{N\cos\theta} + \frac{f_s\sin\theta}{N\cos\theta}}$

$\frac{v^2}{rg} = \frac{\tan\theta - f_s/N}{1 + \frac{f_s}{N}\tan\theta}$

If $V = V_{max}$ & $\frac{f_s}{N} = \mu_s$

$V_{max}^2 = rg\left(\frac{\tan\theta - \mu_s}{1 + \mu_s \tan\theta}\right)$

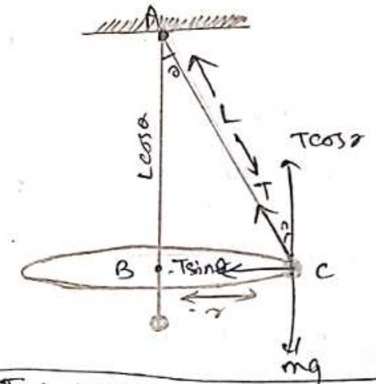
$$\boxed{V_{max} = \sqrt{rg\left(\frac{\tan\theta - \mu_s}{1 + \mu_s \tan\theta}\right)}}$$

(MOImp)

* Define conical pendulum & Drive & expression for its time period.

Conical pendulum :- A simple pendulum in which the string moves along the surface of a cone & the point object perform horizontal circular motion.

⇒ consider a conical pendulum of mass 'm' of length 'L'



The force acting on the bob are

1. mg vertically downward
2. Tension 'T' along the string.
   It resolved into
   (a) vertical → $T\cos\theta$
   (b) horizontal → $T\sin\theta$

mg balance $T\cos\theta$

$\therefore mg = T\cos\theta$ — (1)

$T\sin\theta$ acts as centripital force.

$\frac{mv^2}{r} = T\sin\theta$

∵ $v = r\omega$

$\therefore \frac{mr^2\omega^2}{r} = T\sin\theta$

$mr\omega^2 = T\sin\theta$ — (2)

(2) by (1)

$\frac{T\sin\theta}{T\cos\theta} = \frac{mr\omega^2}{mg}$

$\frac{\sin\theta}{\cos\theta} = \frac{r\omega^2}{g}$ — (*)

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$$\frac{g\sin\theta}{r\cos\theta} = \omega^2$$

$$\omega = \sqrt{\frac{g\sin\theta}{r\cos\theta}} \quad \text{—(i)}$$

∵ In ΔABC.

$$\sin\theta = \frac{opp}{hypo}$$

$$\sin\theta = \frac{r}{L}$$

∴ $r = L\sin\theta$

↳ put in (i)

$$\therefore W = \sqrt{\frac{g\sin\theta}{r\cos\theta}}$$

$$= \sqrt{\frac{g\sin\theta}{L\sin\theta\cdot\cos\theta}}$$

$$\boxed{\omega = \sqrt{\frac{g}{L\cos\theta}}}$$

∵ we know that.

$$\omega = \frac{2\pi}{T}$$

$$\therefore T = \frac{2\pi}{\omega}$$

$$\therefore T = \frac{2\pi}{\sqrt{\frac{g}{L\cos\theta}}}$$

$$\boxed{T = 2\pi\sqrt{\frac{L\cos\theta}{g}}}$$

Since., frequency, $n = \frac{1}{T}$

$$\therefore n = \frac{1}{2\pi\sqrt{\frac{L\cos\theta}{g}}}$$

$$\boxed{\therefore n = \frac{1}{2\pi}\sqrt{\frac{g}{L\cos\theta}}}$$

from (*)

$$\frac{\sin\theta}{\cos\theta} = \frac{r\omega^2}{g}$$

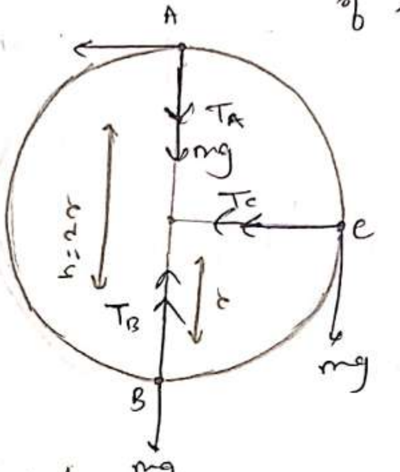
$$\tan\theta = \frac{r\omega^2}{g}$$

$$\therefore \omega^2 = \frac{g\tan\theta}{r}$$

$$\boxed{\omega = \sqrt{\frac{g\tan\theta}{r}}}$$

*** Drive an expression for velocities at different position when a particle is performing vertical circular motion :**

⇒ consider a body of mass 'm' attached to one end of a string whirled in a vertical circle of radius 'r'.



At any position of the body. two forces are acting on it.

① its weight 'mg' vertically downward.

② force due to tension along the string towards the center.

→ At uppermost position (A) :-

mg & T_A necessary provides centripetal force.

$$\therefore mg + T_A = \frac{mv_A^2}{r}$$

at highest position 'A' to maintain circular motion tension T_A should be minimum.

∴ $T_A = 0$

$$\therefore mg + 0 = \frac{mv_A^2}{r}$$

$$\cancel{m}g = \frac{\cancel{m}v_A^2}{r}$$

$$rg = v_A^2 \quad \text{—①}$$

$$\boxed{V_A = \sqrt{rg}} \quad \text{—(A)}$$

→ ~~Velocity at lowest point (B) :-~~

At lowest point 'B'

By principle of conservation of energy

from B to A   from A to B

change in K.E = change in PE

(ΔKE)   (ΔP)

$$\frac{1}{2}mV_B^2 - \frac{1}{2}mV_A^2 = mgh$$

$$\frac{1}{2}\cancel{m}(V_B^2 - V_A^2) = \cancel{m}g2r \quad \text{...(h=2r)}$$

$$\frac{1}{2}(V_B^2 - V_A^2) = 2rg$$

$$V_B^2 - V_A^2 = 4rg$$

$$V_B^2 - rg = 4rg \quad \text{...(from ①)}$$

$$V_B^2 = 4rg + rg.$$

$$V_B^2 = 5rg \quad \text{—①}$$

$$\boxed{V_B = \sqrt{5rg}} \quad \text{—(B)}$$

(3) at midway point (C) :-

At midway point C :

By principle of conservation of energy.

Change in kinetic energy (ΔKE) = change in potential energy (ΔP)

$$\frac{1}{2} mV_c^2 - \frac{1}{2} mV_B^2 = mgh$$

$$\frac{1}{2} m (V_c^2 - V_B^2) = mg2r \quad \cdots (h = 2r)$$

$$\frac{1}{2} (V_c^2 - V_B^2) = 2rg$$

$$\frac{1}{2} V_c^2 - 5rg = 2rg$$

$$\frac{1}{2} mV_c^2 - \frac{1}{2} mV_A = mgh - mgr$$

$$\frac{1}{2} m (V_c^2 - V_A^2) = 2mrg - mrg$$

$$\frac{1}{2} m (V_c^2 - V_A^2) = mrg$$

$$V_c^2 - V_A^2 = 2rg$$

$$V_c^2 - rg = 2rg \quad \cdots (\text{from } 1)$$

$$V_c^2 = 2rg + rg$$

$$V_c^2 = 3rg$$

$$\boxed{V_c = \sqrt{3rg}}$$

other method.

$$\Delta KE = \Delta PE$$

$$\frac{1}{2} mV_B^2 - \frac{1}{2} mV_c^2 = mgh$$

$$\frac{1}{2} m (V_B^2 - V_c^2) = mrg \quad \cdots (h = r)$$

$$\frac{1}{2} (V_B^2 - V_c^2) = rg$$

$$V_B^2 - V_c^2 = 2rg$$

$$5rg - V_c^2 = 2rg$$

$$5rg - 2rg = V_c^2$$

$$\boxed{V_c = \sqrt{3rg}}$$

* Prove that the tension bet the top position & bottom position is equal to 6mg.

at top position.

$$T_A + mg = \frac{mV_A^2}{r} \quad \text{— (1)}$$

at bottom position, $T_B - mg = \frac{mV_B^2}{r}$ — (2)

Substract (2) by (1)

$$T_B - mg - (T_A + mg) = \frac{mV_B^2}{r} - \frac{mV_A^2}{r}$$

$$T_B - mg - T_A - mg = \frac{m}{r} (V_B^2 - V_A^2)$$

$$T_B - T_A - 2mg = \frac{m}{r} (V_B^2 - V_A^2)$$

$$T_B - T_A - 2mg = \frac{m}{r} (5rg - rg)$$

$$T_B - T_A - 2mg = \frac{m}{r} (4rg)$$

$$T_B - T_A - 2mg = 4mg$$

$$T_B - T_A = 4mg + 2mg$$

$$\boxed{T_B - T_A = 6mg}$$

Hence proved.

* Tension at any position *

$$\boxed{T_P = \frac{mV^2}{r} + mg\cos\theta}$$

$$T_A = \frac{mV_A^2}{r} + mg$$

$$T_B = \frac{mV_B^2}{r} - mg$$

$$T_c = \frac{mV_c^2}{r}$$

* velocity at any position *

$$\boxed{V_P = \sqrt{rg(3 + 2\cos\theta)}}$$

$$\because V_c = \sqrt{rg(3 + 2\cos\theta)}$$

here $\theta = 90°$

$\therefore \cos 90 = 0$

$$\therefore V_c = \sqrt{rg(3 + 2\cos(90))}$$

$$= \sqrt{rg(3 + 2 \times 0)}$$

$$= \sqrt{rg(3 + 0)}$$

$$\boxed{V_c = \sqrt{3rg}}$$

\# moment of Inertia \#

→ Define moment of inertia state its units, dimensions & physical significance.

Moment of inertia (I) :- It is define as the sum of product of mass of each particles & square of the distance from axis of rotation is called "moment of inertial".

$$\boxed{I = \sum_{i=1}^{n} m_i r_i^2}$$

SI unit → kgm^2

Dimension → $[L^2 M^1 T^0]$

Physical significance :-

(i) moment of inertia depends upon distribution of mass on the body.

(ii) moment of inertial depends upon orientation of axis of rotation

(iii) Distance of mass from axis of rotation.

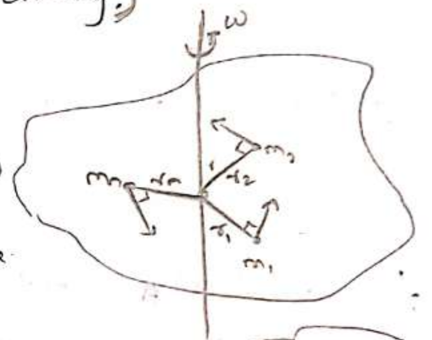
\# Drive an expression for K.E of rotating body :

(consider a rigid body of mass 'M' rotating about an axis passing through its center & $\perp^{lar}$ to its plane with constant angular velocity 'ω'.

Let, the body be devided int 'n' no. of particles each of mass $m_1, m_2 \ldots m_n$ situated at a distance of $r_1, r_2 \ldots r_n$ from axis of rotation when body rotates each particle perform circular motion with diff radii & velocity.)

⇒ linear velocity of each particle is given by

$v_1 = r_1\omega$, $v_2 = r_2\omega$ .... $v_n = r_n\omega$



Kinetic energy of each particle

$$K.E_1 = \frac{1}{2}mv^2 = \frac{1}{2}m(r_1\omega)^2$$

$$K.E_1 = \frac{1}{2}m_1 r_1^2\omega^2$$

$$KE = \frac{1}{2}mv^2$$
$$v = r\omega$$
$$KE = \frac{1}{2}mr^2\omega^2$$

Similarly, $K.E_2 = \frac{1}{2}m_2 r_2^2\omega^2$

⋮

$$KE_n = \frac{1}{2}m_n r_n^2\omega^2$$

∴ Total K.E of rotating body is given by,

$$(K.E)_{Total} = K.E_1 + K.E_2 + \ldots + K.E_n$$

$$= \frac{1}{2}m_1r_1^2\omega^2 + \frac{1}{2}m_2r_2^2\omega^2 + \ldots + \frac{1}{2}m_nr_n^2\omega^2$$

$$= \frac{1}{2}\left[m_1r_1^2 + m_2r_2^2 + \ldots + m_nr_n^2\right]\omega^2$$

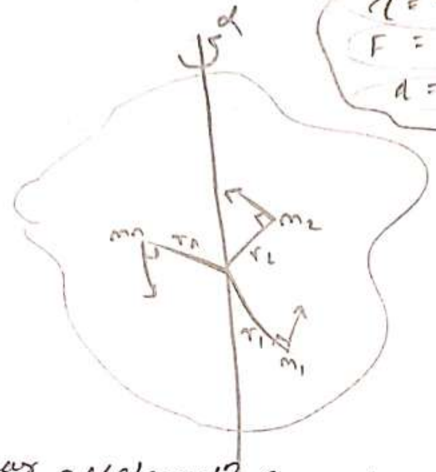
$$\boxed{(K.E)_{Total} = \frac{1}{2}I\omega^2}$$

(4) * Derive an exp. for torque acting on a rotating body?

$\tau = \bar{r} \times \bar{F}$

$F = ma$

$a = r\alpha$



Linear accelerat^n of each particles, is given by,

$a_1 = r_1\alpha$

$a_2 = r_2\alpha$

⋮

$a_n = r_n\alpha$

Force acting on each particles is given by

$F_1 = m_1a_1 = m_1r_1\alpha$

$F_2 = m_2a_2 = m_2r_2\alpha$

⋮

$F_n = m_na_n = m_nr_n\alpha$

$\bar{\tau} = \bar{r} \times \bar{F}$

$= \bar{r}F\sin\theta \quad \ldots (A \times B = AB\sin\theta)$

Here, $\theta = 90$ ∴ $\sin\theta = 1$

$\boxed{\therefore \tau = rF}$

Torque acting on each particle is given by

$\tau_1 = r_1F_1$
$= r_1m_1r_1\alpha$
$= m_1r_1^2\alpha$

$\tau_2 = r_2F_2$
$= r_2m_2r_2\alpha$
$= m_2r_2^2\alpha$

$\ldots \tau_n = r_nF_n$
$= m_nr_n^2\alpha$

Total torque acting on the body is given by

$\tau = \tau_1 + \tau_2 + \cdots \tau_n$

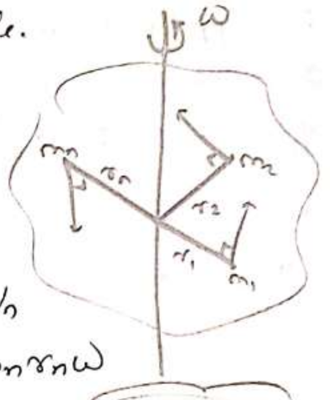
$= m_1r_1^2\alpha + m_2r_2^2\alpha + \cdots m_nr_n^2\alpha$

$= [m_1r_1^2 + m_2r_2^2 + \cdots m_nr_n^2]\alpha$

$$\boxed{\tau = I\alpha}$$

* Derive an exp. for angular momentum of rotating body?

Linear velocity of each particle is given by,



$v_1 = r_1\omega \quad v_2 = r_2\omega \ldots v_n = r_n\omega$

Linear momentum of each particle is given by,

$P_1 = m_1v_1 \quad P_2 = m_2v_2 \ldots P_n = m_nv_n$

$P_1 = m_1r_1\omega \quad P_2 = m_2r_2\omega \ldots P_n = m_nr_n\omega$

$L = rp$

$p = mv$

$v = r\omega$

We know that.

$\bar{L} = \bar{r} \times \bar{p}$

$L = rp\sin\theta$

Here $\theta = 90°$ ∴ $\sin 90 = 1$

$\boxed{\therefore L = rp}$

∴ ~~Total~~ angular momentum of each particle ~~rotating body~~ is given by

$L_1 = r_1P_1$
$= r_1m_1r_1\omega$
$= m_1r_1^2\omega$

$L_2 = r_2P_2$
$= r_2m_2r_2\omega$
$= m_2r_2^2\omega$

$\ldots L_n = r_nP_n$
$= r_nm_nr_n\omega$
$= m_nr_n^2\omega$

Total angular momentum of rotating body is given by

$L_{body} = L_1 + L_2 + \cdots L_n$

$= m_1r_1^2\omega + m_2r_2^2\omega + \cdots + m_nr_n^2\omega$

$= [m_1r_1^2 + m_2r_2^2 + \cdots + m_nr_n^2]\omega$

$$\boxed{L = I\omega}$$

* Define radius of gyration of a rigid body & state its physical significance.

Radius of gyration :-

Radius of gyration of a rigid body about an axis of rotation is defined as distance between axis of rotation & a point where whole mass of a body is suppose to be concentrated so as to get same moment of inertia (M.I) as about given axis.

$I = mr^2$

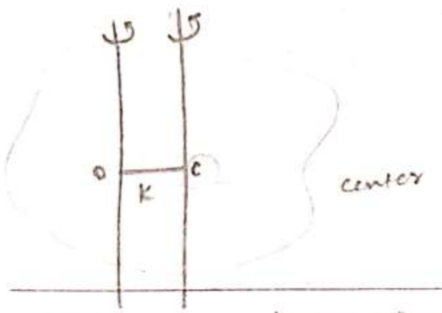
$I = mK^2$

$K^2 = \frac{I}{M}$

$$\boxed{K = \sqrt{\frac{I}{M}}}$$

SI unit → m

Dimensions → $[L^1]$



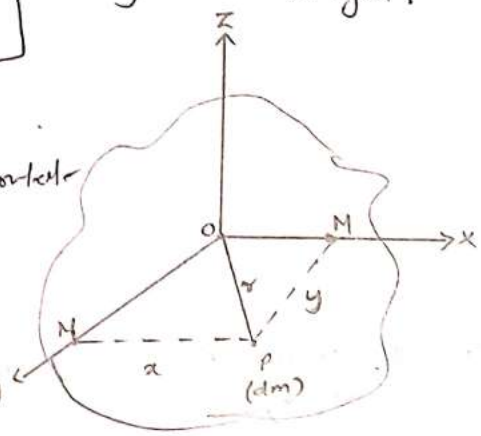
Physical significance :-

Same as moment of inertia → {pg-3}

\# State & prove therom of perpendicular axes \# ✱✱✱

Statement :- " Moment of inertia of a plane lemina about a $\perp^{lar}$ axis is equal to sum of moment of inertia about two mutually $\perp^{lar}$ axes in the plane of lemina having same origin.

$$\boxed{\therefore I_z = I_x + I_y}$$



proof :-

consider a lemina in horizontal xyz plane.

Let I_x, I_y & I_z are the moment of inertia about x, y, z axes respectively.

$OM = NP = x$ — ①

$ON = MP = y$ — ②

$I_z = \int r^2 . dm$

$I_x = \int y^2 . dm$

$I_y = \int x^2 dm$

In $\triangle$ OMP

$OP^2 = OM^2 + MP^2$

$r^2 = y^2 + x^2$

multiply throughout by 'dm' & taking integration.

$\int r^2 . dm = \int y^2 . dm + \int x^2 . dm$

$\because I_z = I_y + I_x$

$$\boxed{\therefore I_z = I_x + I_y}$$

Hence proved.

(3) Theorem of parallel axes ***

Statement :-

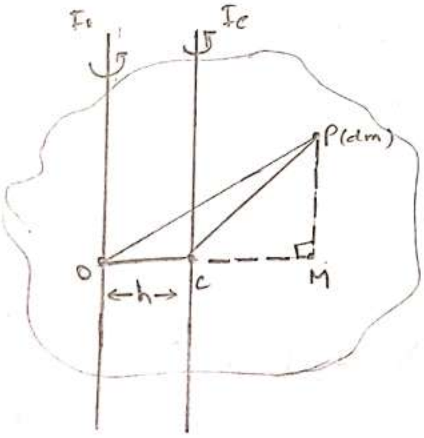
"The moment of inertia about any axis is equal to the sum of its moment of inertia about a parallel axis passing through its centre of mass and the product of the mass of the body and square of the $\perp$lar distⁿ betⁿ parallel axis."

$$\boxed{I_o = I_c + Mh^2}$$

$I_o \rightarrow$ moment of inertia about an axis passing through O.

$I_c \rightarrow$ moment of inertia about an axis passing through center of mass C.

$h \rightarrow$ perpendicular distⁿ betⁿ parallel axis.



In ΔPMC

$$PC^2 = CM^2 + PM^2 \quad —(1)$$

In ΔOMP.

$$OP^2 = PM^2 + MO^2$$

$$= PM^2 + (OC + CM)^2$$

$$\cancel{= PM^2 + CM^2 + OC^2}$$

$$= PM^2 + OC^2 + CM^2 + 2\,OC \cdot CM$$

$$= (PM^2 + CM^2) + OC^2 + 2\,OC \cdot CM$$

$$= PC^2 + OC^2 + 2 \cdot OC \cdot CM$$

$$OP^2 = PC^2 + 2 \cdot OC \cdot CM + OC^2$$

multiply by 'dm' & taking integration.

$$\int OP^2 \cdot dm = \int PC^2 \cdot dm + 2\int OC \cdot CM\,dm + \int OC^2 \cdot dm \quad —(2)$$

$\because$ we know that,

$$\int OP^2 \cdot dm = I_o$$

$$\int PC^2 \cdot dm = I_c$$

$$\int dm = M$$

$$OC = h$$

$$\int CM = 0$$

$\therefore$ eqⁿ (2) becomes

$$I_o = I_c + 2\,h \int CM\,dm + \int h^2 \cdot dm$$

$$I_o = I_c + 2 \times h \times 0 + h^2 \cdot \int dm$$

$$I_o = I_c + 0 + h^2 M$$

$$\boxed{I_o = I_c + Mh^2}$$

Hence proved.

* State & prove principle of conservation of angular momentum : ***

Statement :-

The angular momentum of the body remain constant if the resultant external torque acting on the body is zero.

Proof :-

we know that,

$$\bar{L} = \bar{r} \times \bar{P}$$

diff w.r.t 't'

$$\frac{d\bar{L}}{dt} = \frac{d(\bar{r} \times \bar{P})}{dt}$$

$$\frac{d\bar{L}}{dt} = \bar{r} \times \frac{d\bar{P}}{dt} + \bar{P} \frac{d\bar{r}}{dt}$$

$$\frac{d\bar{L}}{dt} = \bar{r} \times \bar{F} + \bar{P} \times \bar{v}$$

$$\left\{ \because \frac{d\bar{P}}{dt} = \bar{F} \quad , \frac{d\bar{r}}{dt} = \bar{v} \right\}$$

$$\frac{d\bar{L}}{dt} = \bar{\tau} + m\bar{v} \times \bar{v}$$

$$\left\{ \because \bar{r} \times \bar{F} = \bar{\tau} \quad P = m\bar{v} \right\}$$

$$\frac{d\bar{L}}{dt} = \bar{\tau} + m(0) \quad \ldots \{\bar{v} \times \bar{v} = 0\} \ldots \{ i \times i = j \times j = k \times k = 0$$

$$\therefore \frac{d\bar{L}}{dt} = \bar{\tau}$$

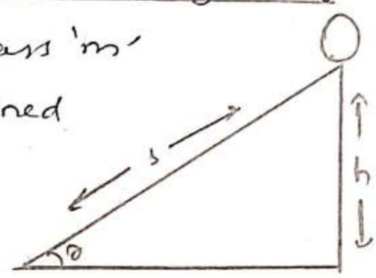
If $\bar{\tau} = 0$

$$\boxed{\therefore \frac{d\bar{L}}{dt} = 0}$$

$\therefore$ Angular momentum is conserved.

* Drive an exp for K.E of a rolling body *

$\Rightarrow$ consider a body of mass 'm' rolling down an inclined Plane from height 'h' inclined at an angle 'θ'.



When the body is rolling down it's perform ① Linear motion ② Rotational motion.

$\therefore$ kinetic energy of rolling body $= (K.E)_{linear} + (K.E)_{rotat^n}$

$$= \frac{1}{2} mv^2 + \frac{1}{2} I\omega^2$$

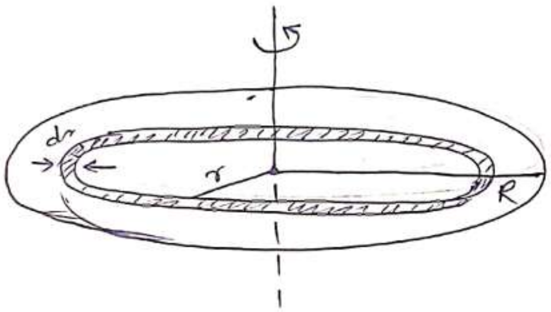
$$= \frac{1}{2} mv^2 + \frac{1}{2} mk^2\omega^2 \quad \{ \because I = mk^2 \}$$

$$= \frac{1}{2} mv^2 + \frac{1}{2} mk \frac{v^2}{R^2} \quad \ldots \left\{ \because v = r\omega \text{ Here } V = R\omega \quad \boxed{\omega = \frac{V}{R}} \right.$$

$$\boxed{\therefore (K.E)_{Rolling} = \frac{1}{2} mV^2 \left[1 + \frac{k^2}{R^2} \right]}$$

⑥ * Drive an exp. for moment of inertia of a uniform disk.?

⇒ consider a uniform disk of mass 'M' & radius 'R' rotating about its own axis.



Surface density (σ) = $\frac{mass}{vol^m} = \frac{M}{\pi R^2}$ — ①

consider one consentric ring of 'dm' of width 'dr' show by Shaded portion.

∴ Area of this ring (A) = $\underset{l}{2\pi r} \cdot \underset{b}{dr}$

$\therefore \sigma = \frac{dm}{2\pi r \cdot dr}$

$\therefore dm = \sigma \times 2\pi r dr$

∴ Moment of inertia of strip ring (I_r) = $\boxed{dm \cdot r^2}$

Moment of inertia of Disc (I) = $\int_0^R I_r$

$\therefore I = \int_0^R I_r$

$= \int_0^R dm\, r^2$

$\therefore I = \int_0^R 2\pi r \cdot dr \times \sigma \times r^2$

$= 2\pi\sigma \int_0^R r^3 dr$

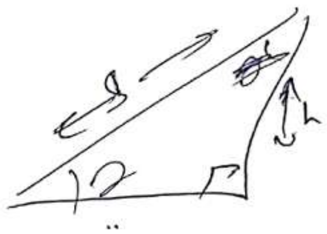
$= 2\pi\sigma \left[\frac{r^4}{4}\right]_0^R$

$= \cancel{2\pi} \times \frac{M}{\cancel{\pi} R^2}\left[\frac{R^4}{\cancel{4}_2} - 0\right]$ ... from ①

$= \frac{M}{2R^2} \times R^4$

$= \frac{MR^2}{2}$

$\boxed{I = \frac{1}{2} MR^2}$



when the sphere rolls down its potential energy is converted into by K.E rolling

∴ By principle of conservation of P.E = (K.E)rolling

$mgh = \frac{1}{2} m v^2 \left(1 + \frac{K^2}{R^2}\right)$

$2gh = v^2\left(1 + \frac{K^2}{R^2}\right)$

$\frac{2gh}{\left(1 + \frac{K^2}{R^2}\right)} = v^2$

$\boxed{\therefore v = \sqrt{\frac{2gh}{\left(1 + \frac{K^2}{R^2}\right)}}}$

For acceler

By Newtons III kinematical eqn

$v^2 = u^2 + 2as$

$u = 0$

$\therefore v^2 = 2as$

$\frac{\cancel{2}gh}{1 + \frac{K^2}{R^2}} = \cancel{2}as$

$\frac{gh}{\left(1 + \frac{K^2}{R^2}\right)} = as$

~~sin θ = opp/hypo = h/s~~

~~∴ s = h sinθ~~

$\sin\theta = \frac{opp}{hypo} = \frac{h}{s}$

$\frac{g\left(\frac{h}{s}\right)}{\left(1 + \frac{K^2}{R^2}\right)}$

$\boxed{\frac{g\sin\theta}{\left(1 + \frac{K^2}{R^2}\right)} = a}$